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Fracture filling fluids identification using azimuthally elastic impedance based on rock physics

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ABSTRACT

Fracture filling fluids identification is an important aspect of fractured reservoirs description. It is well known that fracture weaknesses parameters (the normal and tangential weaknesses) are closely related to fracture parameters (fracture density, fracture aspect ratio and fracture fillings). Hence, the fracture filling fluids can be identified after the fracture weaknesses parameters being obtained. The efficient method, which is presented to estimate the fracture weaknesses parameters in this paper, is azimuthally elastic impedance (EI) inversion. We first derive a new azimuthal EI equation which contains fracture weaknesses parameters. Using the formula, we propose a method of seismic inversion for elastic parameters (P-wave and S-wave impedances) and fracture weaknesses parameters. The fluid factor, which is introduced to identify the fluids in fractures, can be estimated after the calculation of elastic and fracture weaknesses parameters. Tests on real data show that the method is more stable and reasonable by reducing the uncertainty for the estimation of elastic parameters and fracture weaknesses parameters. The inverted results are in good agreement with the results of well log data.

1. Introduction

Underground fractures prediction and fluid identification are two essential aspects in carbonate rock reservoir and unconventional reservoirs (shale gas, tight gas and tight oil reservoirs). The estimation of elastic parameters and fracture weaknesses parameters is very important for the fractured reservoirs description (Liu and Martinez, 2012). In this paper, we aim to explore the efficient method which utilizes azimuthally pre-stack seismic data to estimate these parameters.

There are two commonly used fracture rock physics models. One is penny-shaped model for cracked media (Hudson, 1980). The other is linear slip model for fractured media (Schoenberg and Douma, 1988). Bakulin et al. (2000) studied the relationship between penny-shaped model and linear slip model. They also presented the equation to describe fracture rock physics parameters (the normal weakness, Δ_N, and the tangential weakness, Δ_T) variation with fracture parameters (fracture density, fracture aspect ratio and fracture fillings), and fracture parameters can be estimated by this equation. Schoenberg and Sayers (1995) introduced the definition of fracture fluid factor which is effective for fluid identification in fractures, and they also pointed out that fracture fluid factor is related to the S-wave to P-wave velocity ratio and fracture weaknesses parameters. Using linear slip model, Sil (2013) estimated fracture parameter from well-log data.

Connolly (1999) first proposed the concept of elastic impedance (EI). EI inversion is a useful tool for estimating elastic parameters. The reservoir rock which is pervaded by vertically and sub-vertically aligned fractures may display horizontal transverse isotropy (HTI). Ruger (1996, 1997, 1998) derived the reflection coefficient approximate formula for a HTI medium. The formula can be approximately described as a function of incident angle and azimuthal angle (Amplitude variation with incident and azimuthal angle, AVAZ), and it provides an opportunity to estimate elastic parameters, anisotropic parameters and fluid indicator of fractured layers from seismic data (Chen et al., 2013; Huang, 2013; Mallick et al., 1998; Shaw and Sen, 2006). It is well known that seismic amplitude response will be more complicated when the parameters of fracture sets are complex (Downton, 2006; Downton and Roure, 2010). In this paper, we aim to study a more effective method to estimate elastic parameters, fracture weaknesses and fracture fluid factor by using EI inversion. However, in HTI media, the present isotropic EI equation is not applicable. Hence, it is very important to derive an equation which is suitable for vertically fractured reservoirs. Martins (2006) proposed EI equation for HTI media. And he also studied the characteristic of EI variation with incident and azimuthal angle. However, Martins' equation is not well used in seismic inversion for elastic parameters because of its complexity. In this paper, we derive a new azimuthal EI equation which contains fracture weaknesses parameters. We also present the inversion method based on the new azimuthal EI equation. Using this method, we can obtain P-wave impedance, S-wave impedance, the normal weakness and the tangential weakness from azimuthal seismic datasets.

There are three main steps for elastic parameters, fracture weaknesses parameters, and fluid factor estimation with the azimuthal EI equation; azimuthal EI inversion, elastic parameters and fracture

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Table 1
Physical properties of an interface separating an isotropic medium from a fractured medium.

		α (km/s)	β (km/s)	ρ (g/cc)	Δ_N	Δ_T
Isotropic layer		6.10	3.40	2.25	0	0
Vertically Fractured	Gas filled	6.10	3.40	2.25	0.6041	0.2142
layer	Oil filled				0.2277	0.2277

parameters extraction, and fluid factor estimation. The azimuthal EI datasets in different incident and azimuthal angles range can be calculated reasonably with a conventional post-stack seismic inversion algorithm. However, the method to estimate the elastic parameters and fracture weaknesses parameters from azimuthal EI datasets is still challenging. On the basis of previous studies, we propose the azimuthal seismic inversion method to obtain the azimuthal EI datasets. P-wave impedance, S-wave impedance and fracture weaknesses parameters are extracted from azimuthal EI datasets then adopt the damped least squares algorithm. The proposed azimuthal EI inversion method renders the elastic and fracture weaknesses parameter estimation from azimuthal EI datasets more stable. Then the fracture fluid factor can be calculated using the inverted elastic parameters and the weaknesses parameters. We will initially derive the novel azimuthal EI equation which contains fracture weaknesses parameters and study the characteristic of azimuthal EI, and then discuss the method to estimate P-wave impedance, S-wave impedance and fracture weaknesses parameters from inverted azimuthal EI datasets. We end with real data case studies that illustrate the method.

2. Theory

2.1. Linear slip model for fractured model

The stiffness matrix of fractured rock can be calculated by using linear slip model (Schoenberg and Sayers, 1995).

$$\mathbf{C}=\begin{bmatrix} M(1-\Delta_N) & \lambda(1-\Delta_N) & \lambda(1-\Delta_N) & 0 & 0 & 0 \\ \lambda(1-\Delta_N) & M\left(1-\left(\frac{\lambda}{M}\right)^2\Delta_N\right) & \lambda\left(1-\frac{\lambda}{M}\Delta_N\right) & 0 & 0 & 0 \\ \lambda(1-\Delta_N) & \lambda\left(1-\frac{\lambda}{M}\Delta_N\right) & (\lambda+2\mu)\left(1-\left(\frac{\lambda}{M}\right)^2\Delta_N\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu(1-\Delta_T) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu(1-\Delta_T) \end{bmatrix} \quad (1)$$

In the equation, M is P-wave modulus, $M = \lambda + 2\mu$. λ and μ are the isotropic Lame's parameters. Δ_N and Δ_T are the normal and tangential weaknesses parameters in the linear slip model.

The relationship between penny-shaped crack model and the linear slip model is:

$$\Delta_N=\frac{4e}{3g(1-g)\left[1+\frac{1}{\pi(1-g)}\left(\frac{k'+4/3\mu'}{\mu a}\right)\right]} \quad (2a)$$

$$\Delta_T=\frac{16e}{3(3-2g)\left[1+\frac{4}{\pi(3-2g)}\left(\frac{\mu'}{\mu a}\right)\right]} \quad (2b)$$

where, $g = \mu / (\lambda + 2\mu) = \beta^2 / \alpha^2$. e is the fracture density, and a is aspect ratio, k' and μ' are the parameters of fillings in the fracture.

2.2. Azimuthal EI equation containing fracture weaknesses parameters

Ruger(1996) derived the reflection coefficient equation for HTI media.

$$R_{PP}(\theta,\phi)=\frac{1}{2}\frac{\Delta Z}{\bar{Z}}+\frac{1}{2}\left\{\frac{\Delta\alpha}{\bar{\alpha}}-\left(\frac{2\bar{\beta}}{\bar{\alpha}}\right)^2\frac{\Delta G}{\bar{G}}+\left[\Delta\delta^{(V)}+2\left(\frac{2\bar{\beta}}{\bar{\alpha}}\right)^2\Delta\gamma\right]\cos^2\phi\right\}\sin^2\theta+\frac{1}{2}\left\{\frac{\Delta\alpha}{\bar{\alpha}}+\Delta\varepsilon^{(V)}\cos^4\phi+\Delta\delta^{(V)}\sin^2\phi\cos^2\phi\right\}\sin^2\theta\tan^2\theta \quad (3)$$

In the equation, $G = \rho\beta^2$, $Z = \rho\alpha$, θ is incident angle. ϕ is azimuthal angle. α and β are P-wave and S-wave velocities. ρ is density. $\Delta\alpha/\bar{\alpha}$ is P-wave reflection coefficient. $\Delta\delta^{(V)}$, $\Delta\varepsilon^{(V)}$ and $\Delta\gamma$ are the difference between the upper and lower anisotropic layers.

The relationship between Thomsen parameters and weaknesses parameters is studied by Bakulin et al. (2000):

$$\begin{aligned} \varepsilon^{(V)} &= -2g(1-g)\Delta_N \\ \delta^{(V)} &= -2g[(1-2g)\Delta_N+\Delta_T] \\ \gamma &= \frac{\Delta_T}{2} \end{aligned} \quad (4)$$

With reference to the formula which is proposed by Bachrach et al. (2009), we propose the reflection coefficient equation which contains fracture rock physics parameters at small incident angle.

$$R_{PP}(\theta,\phi)=\sec^2\theta R_P-8g\sin^2\theta R_S-\left(g\cos^2\phi\sin^2\theta\right)(1-2g)R_{\Delta_N}+\left(g\cos^2\phi\sin^2\theta\right)R_{\Delta_T}, \quad (5)$$

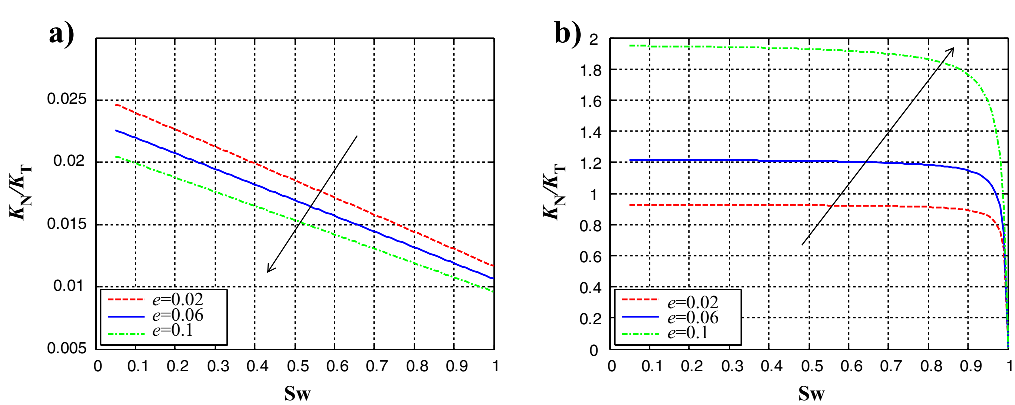

Fig. 1. Fracture fluid factor variation with fracture density and water saturation. (The arrow indicates the increase of fracture density). (a) The fractures are filled with the mixture of oil and water. K_N / K_T gradually reduces with water saturation (Sw) and fracture density e, and the value is around zero. (b) The fractures are filled with the mixture of gas and water. K_N / K_T increases with fracture density, and it shows a decreasing trend with the increase of water saturation. The decrease trend is small when the water saturation is less than 0.9.

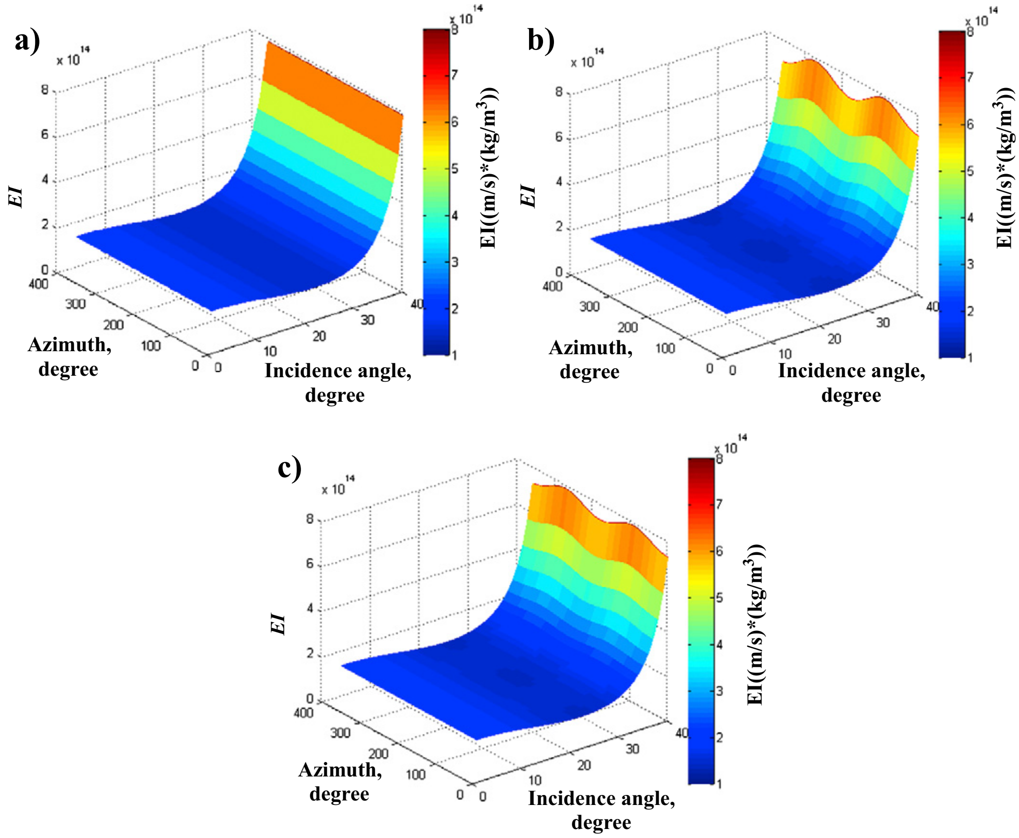

Fig. 2. Azimuthal EI variation with azimuth and incidence angles. (a) Azimuthal EI for isotropic layer. (b) Azimuthal EI for gas-filled fractured layer. (c) Azimuthal EI for oil-filled fractured layer.

where $R_P = \frac{1}{2}\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\rho}{\rho}\right)$, $R_S = \frac{1}{2}\left(\frac{\Delta\beta}{\beta} + \frac{\Delta\rho}{\rho}\right)$, $R_{\Delta_N} = \Delta_{N2} - \Delta_{N1}$, $R_{\Delta_T} = \Delta_{T2} - \Delta_{T1}$, Δ_{N1}, Δ_{N2}, Δ_{T1}, and Δ_{T2} are the fracture weaknesses parameters of the upper and lower layers.

With the assumption that the PP wave reflection coefficient is for an isotropic half space over an isotropic half space, Eq. (5) becomes:

$$R_{PP}(\theta,\phi) = \sec^2\theta R_P - 8g\sin^2\theta R_S - \left(g\cos^2\phi\sin^2\theta\right)(1-2g)\Delta_N + \left(g\cos^2\phi\sin^2\theta\right)\Delta_T \quad (6)$$

Connolly (1999) derived the elastic impedance equation.

$$R_{PP} \approx \frac{\mathrm{EI}_n - \mathrm{EI}_{n-1}}{\mathrm{EI}_n + \mathrm{EI}_{n-1}} \approx \frac{1}{2}\frac{\Delta \mathrm{EI}}{\overline{\mathrm{EI}}} \quad (7)$$

For HTI media, $R_{PP} = R_{PP}(\theta, \phi)$ and $\mathrm{EI} = \mathrm{EI}(\theta, \phi)$.

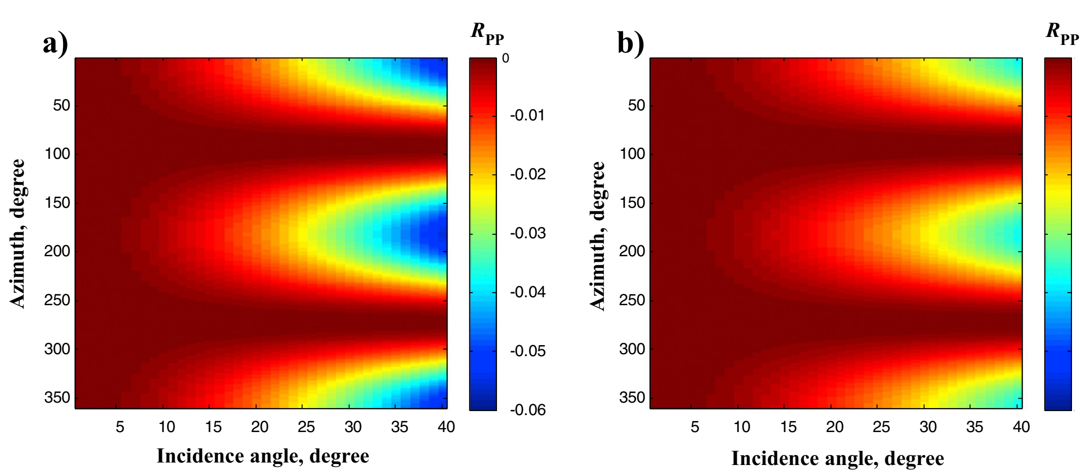

Fig. 3. Reflection coefficients calculated by using azimuthal EI. (a) Reflection coefficients for an isotropic layer overlying gas-filled fractured layer. (b) Reflection coefficients for an isotropic layer overlying oil-filled fractured layer.

Now we present a new azimuthal EI equation which contains fracture weaknesses parameters.

$$\mathrm{EI}(\theta,\phi) = I_{P0}\left(\frac{I_P}{I_{P0}}\right)^{a(\theta)}\left(\frac{I_S}{I_{S0}}\right)^{b(\theta)} \exp\left[c(\theta,\phi)\Delta_N + d(\theta,\phi)\Delta_T\right] \quad (8)$$

where $a(\theta) = \sec^2\theta$, $b(\theta) = -8g\ \sin^2\theta$, $c(\theta,\phi) = -2(g\ \cos^2\phi\ \sin^2\theta)(1-2g)$, $d(\theta,\phi) = 2g\ \cos^2\phi\ \sin^2\theta$.

I_P is P-wave impedance. I_S is S-wave impedance. $I_{P0} = \alpha_0\rho_0$. $I_{S0} = \beta_0\rho_0$. α_0, β_0, and ρ_0 are constants, which are introduced by Whitcombe (2002).

2.3. Azimuthal EI inversion for fracture weaknesses parameters

The estimation of fracture weaknesses parameters includes the extraction of azimuthal EI inversion and the extraction of fracture weaknesses parameters.

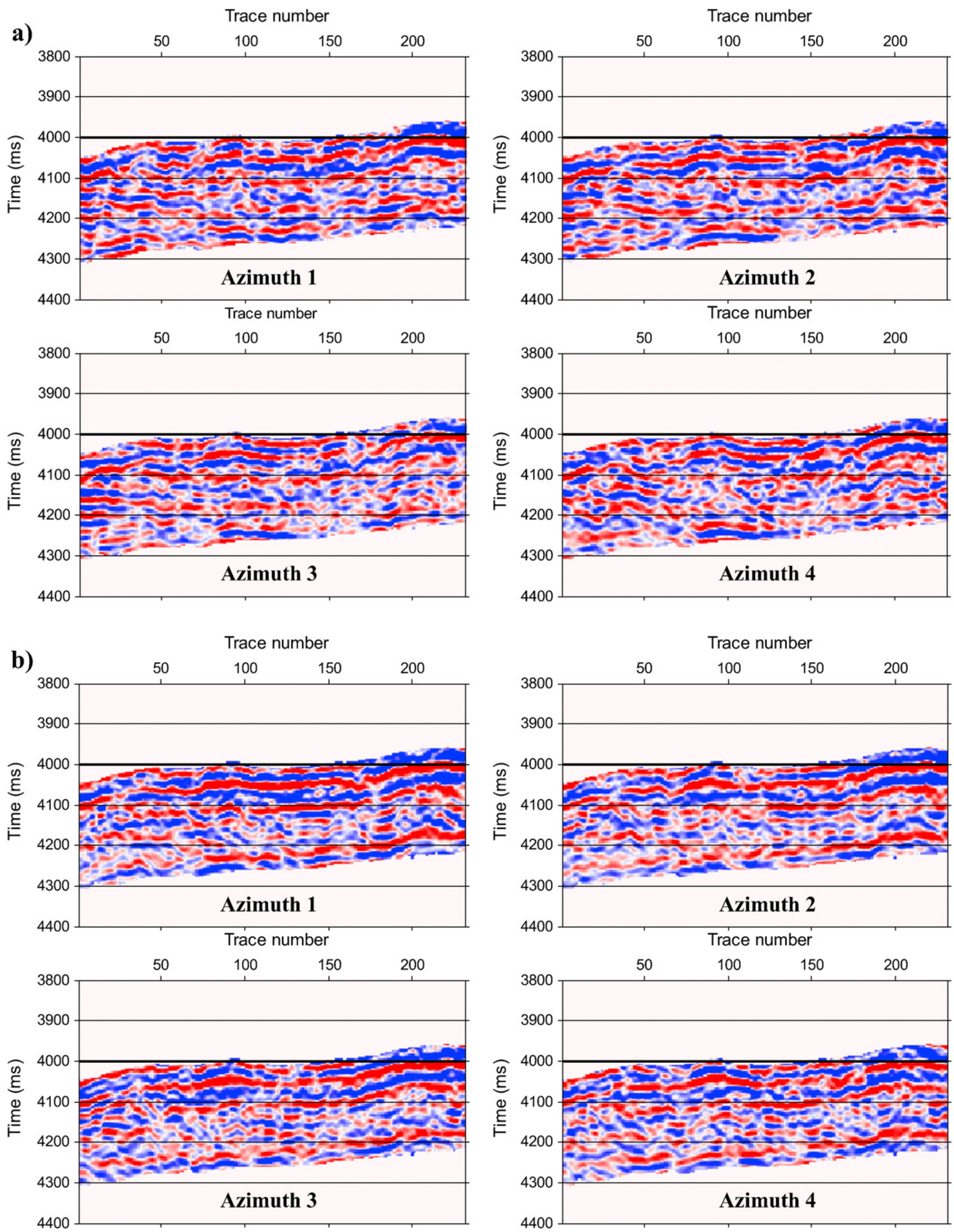

Fig. 4. Angle-stack seismic profiles with different azimuthal angles. (a) 8° (3°–12°) (b) 17° (12°–21°) (c) 26° (21°–30°).

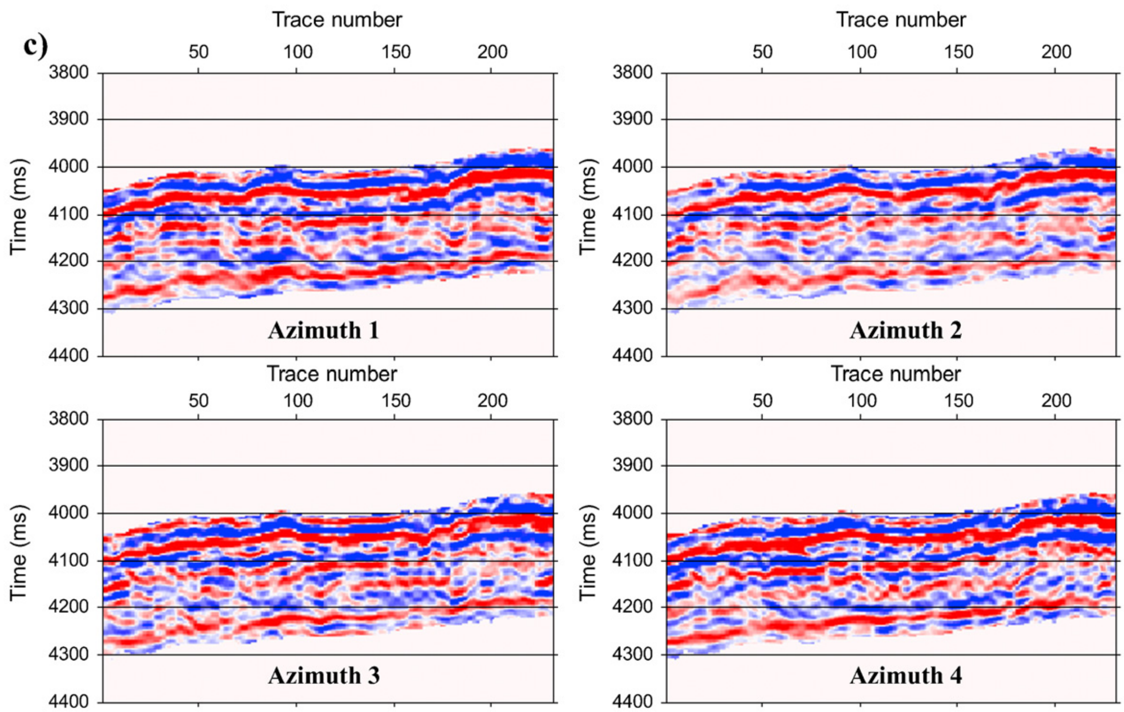

Fig. 4 (*continued*).

The logarithms of azimuthal EI at different incident and azimuthal angles are:

$$\begin{aligned}
\ln\left[\frac{\mathrm{EI}(\theta_1,\phi_1)}{I_{P0}}\right] &= a(\theta_1)\ln\left(\frac{I_P}{I_{P0}}\right)+b(\theta_1)\ln\left(\frac{I_S}{I_{S0}}\right)+c(\theta_1,\phi_1)\Delta_N+d(\theta_1,\phi_1)\Delta_T\\
\ln\left[\frac{\mathrm{EI}(\theta_2,\phi_2)}{I_{P0}}\right] &= a(\theta_2)\ln\left(\frac{I_P}{I_{P0}}\right)+b(\theta_2)\ln\left(\frac{I_S}{I_{S0}}\right)+c(\theta_2,\phi_2)\Delta_N+d(\theta_2,\phi_2)\Delta_T\\
\ln\left[\frac{\mathrm{EI}(\theta_3,\phi_3)}{I_{P0}}\right] &= a(\theta_3)\ln\left(\frac{I_P}{I_{P0}}\right)+b(\theta_3)\ln\left(\frac{I_S}{I_{S0}}\right)+c(\theta_3,\phi_3)\Delta_N+d(\theta_3,\phi_3)\Delta_T
\end{aligned} \tag{9}$$

In order to obtain I_P, I_S, Δ_N and Δ_T, the equation is simplified to

$$d = GX, \tag{10}$$

where

$$d=\begin{pmatrix}\ln\left[\frac{\mathrm{EI}(\theta_1,\phi_1)}{I_{P0}}\right]\\ \ln\left[\frac{\mathrm{EI}(\theta_2,\phi_2)}{I_{P0}}\right]\\ \ln\left[\frac{\mathrm{EI}(\theta_3,\phi_3)}{I_{P0}}\right]\end{pmatrix},\quad X=\begin{bmatrix}ln\left(\frac{I_P}{I_{P0}}\right)\\ \ln\left(\frac{I_S}{I_{S0}}\right)\\ \Delta_N\\ \Delta_T\end{bmatrix}$$

$$G=\begin{bmatrix}a(\theta_1) & b(\theta_1) & c(\theta_1,\phi_1) & d(\theta_1,\phi_1)\\ a(\theta_2) & b(\theta_2) & c(\theta_2,\phi_2) & d(\theta_2,\phi_2)\\ a(\theta_3) & b(\theta_3) & c(\theta_3,\phi_3) & d(\theta_3,\phi_3)\end{bmatrix}.$$

The parameters may be estimated through damped least squares method. The unknown X was derived:

$$X=\left[G^T G+\sigma I\right]^{-1}G^T d, \tag{11}$$

where G^T is transpose of matrix G, σ is damping factor, and I is an identity matrix. The choice of damping factor depends mainly on experiments (Yang, 1997). In the extreme, when there is no noise, σ is zero.

2.4. Fracture fluid identification factor estimation

Schoenberg and Sayers (1995) proposed that K_N/K_T might be applied to indicate the fracture fluid. The definition of K_N/K_T shows as follow:

$$\frac{K_N}{K_T}=\frac{\dfrac{\Delta_N}{(\lambda+2\mu)(1-\Delta_N)}}{\dfrac{\Delta_T}{\mu(1-\Delta_T)}}=g\frac{\Delta_N(1-\Delta_T)}{\Delta_T(1-\Delta_N)} \tag{12}$$

Gas-filled and oil-filled models are built for the description of the characteristics of fracture fluid factor. The physical properties of our models are listed in Table 1.

Fig. 1 shows the variation of K_N/K_T with fracture density and water saturation when fractures are filled with different types of fluid.

From Fig. 1, we can find the variations of K_N/K_T with fracture density and water saturation are remarkably different when fractures are filled with different types of fluid. Using Eq. (12), we can calculate K_N/K_T, after the estimation of elastic parameters and fracture weaknesses parameters by using azimuthal EI inversion.

3. Example

3.1. The characteristic of azimuthal EI

We build two vertically fractured models to study the characteristic of azimuthal EI. The difference between the two models is that the fracture is filled with gas and the other is filled with oil. We assume that the upper layer is isotropic.

Fig. 2 shows us that azimuthal EI for the isotropic and vertically fractured layers changes with incident angle obviously. From Fig. 2b and c, we find that the azimuthal EI of the fractured layer changes with not only the incident angle but also the azimuthal angle.

From Fig. 3, we find that there are a big difference between the reflection coefficients for gas-filled fractures and oil-filled fractures.

3.2. Real data

Real data is used to validate the application of azimuthal EI inversion for elastic parameters and fracture weaknesses parameters. True-amplitude processing has been implemented before inversion of the real data. Azimuthal-angle stack seismic profiles are shown in Fig. 4. The inverted results of azimuthal EI are displayed in Fig. 5. The estimated results of I_P, I_S, Δ_N and Δ_T are displayed in Fig. 6, respectively.

From Fig. 6, we find that the inverted P-wave impedance result at well location (CDP 90) shows low value and the inverted fracture weaknesses parameters show high value in the target fractured reservoir at 4050 ms.

Fig. 7 shows the inverted fracture fluid factor result. From Fig. 7, we can see that the fracture fluid factor at well location (CDP 90) shows high value in the target fractured reservoir at 4050 ms and 4200 ms. This result is consistent with the drilling and rock physics analysis result.

4. Discussion

The above analysis shows how the elastic and anisotropic properties of fractured layers can be obtained using azimuthal EI inversion. The results are P-wave impedance, S-wave impedance, the normal weakness, the tangential weakness and fracture fluid factor. For hydrocarbon-filled fractured layers, P-wave impedance shows low value and the inverted weaknesses parameters show high value in the target reservoir. We also present the estimated fracture fluid factor, which shows high value in gas-filled fractured reservoir.

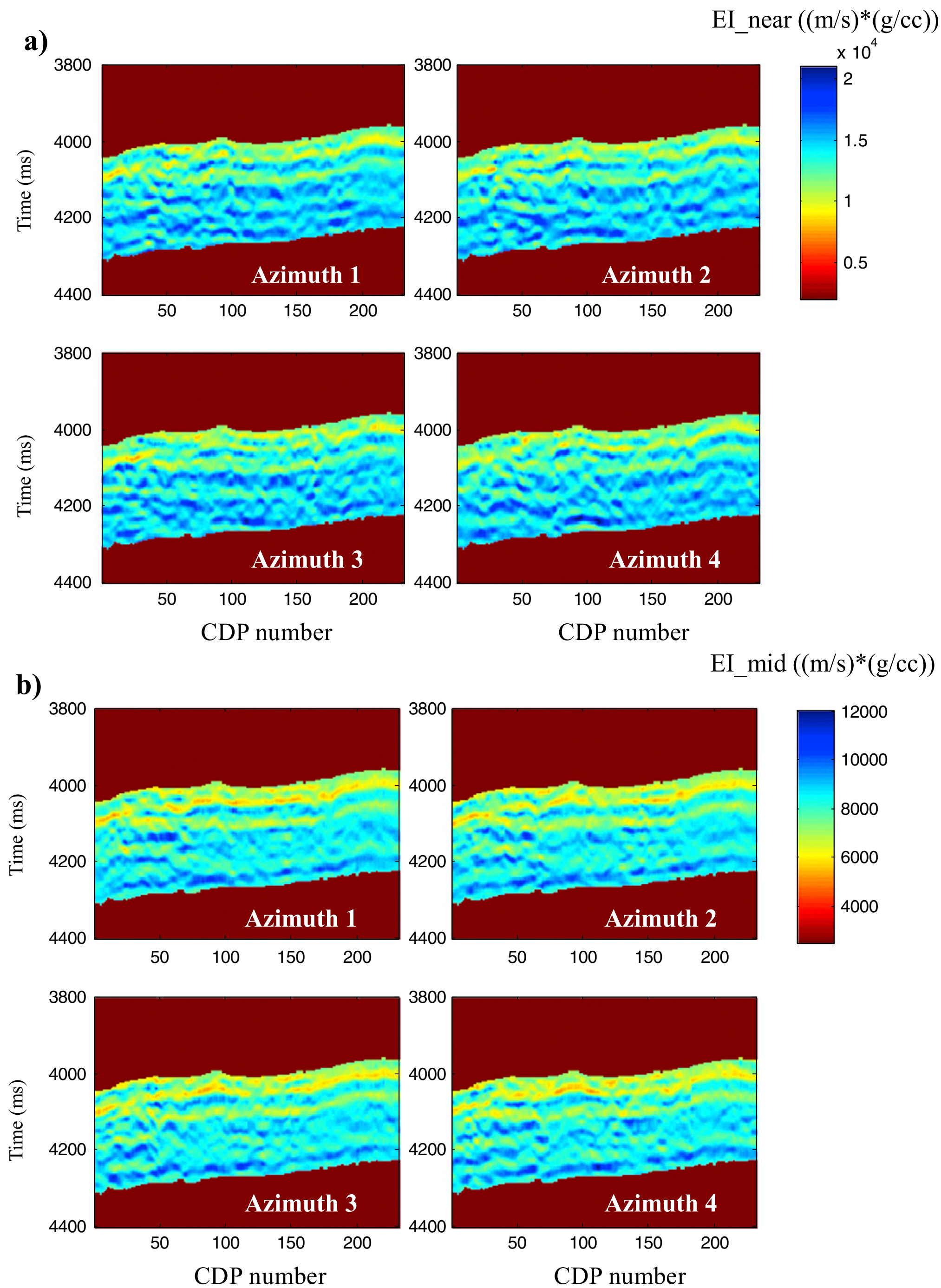

Fig. 5. Azimuthal EI inverted results. (a) 8° (3°–12°) (b) 17° (12°–21°) (c) 26° (21°–30°).

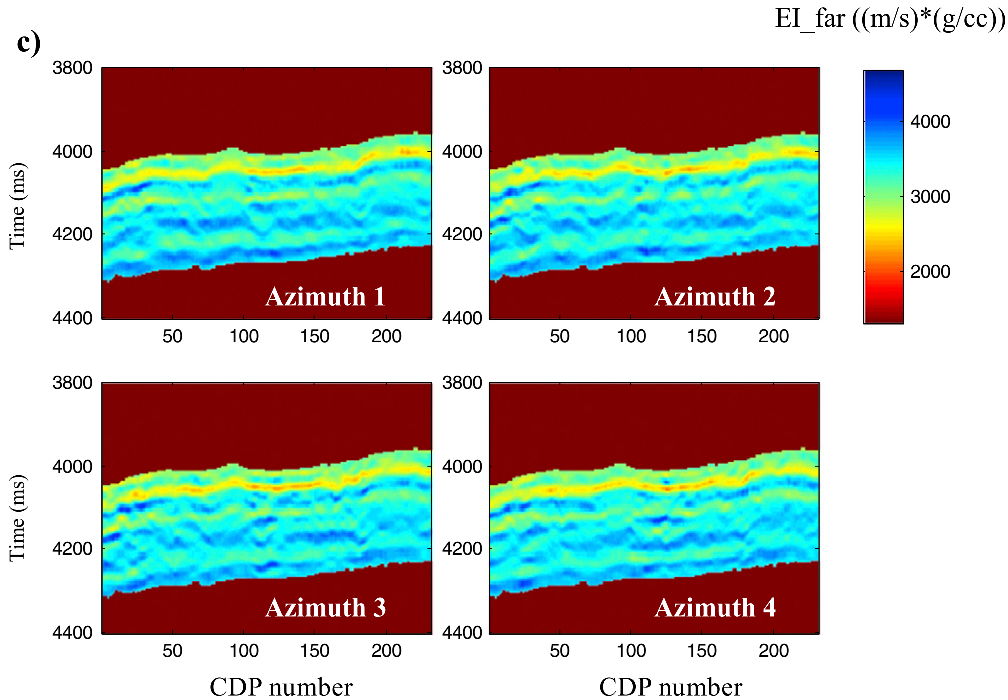

Fig. 5 (*continued*).

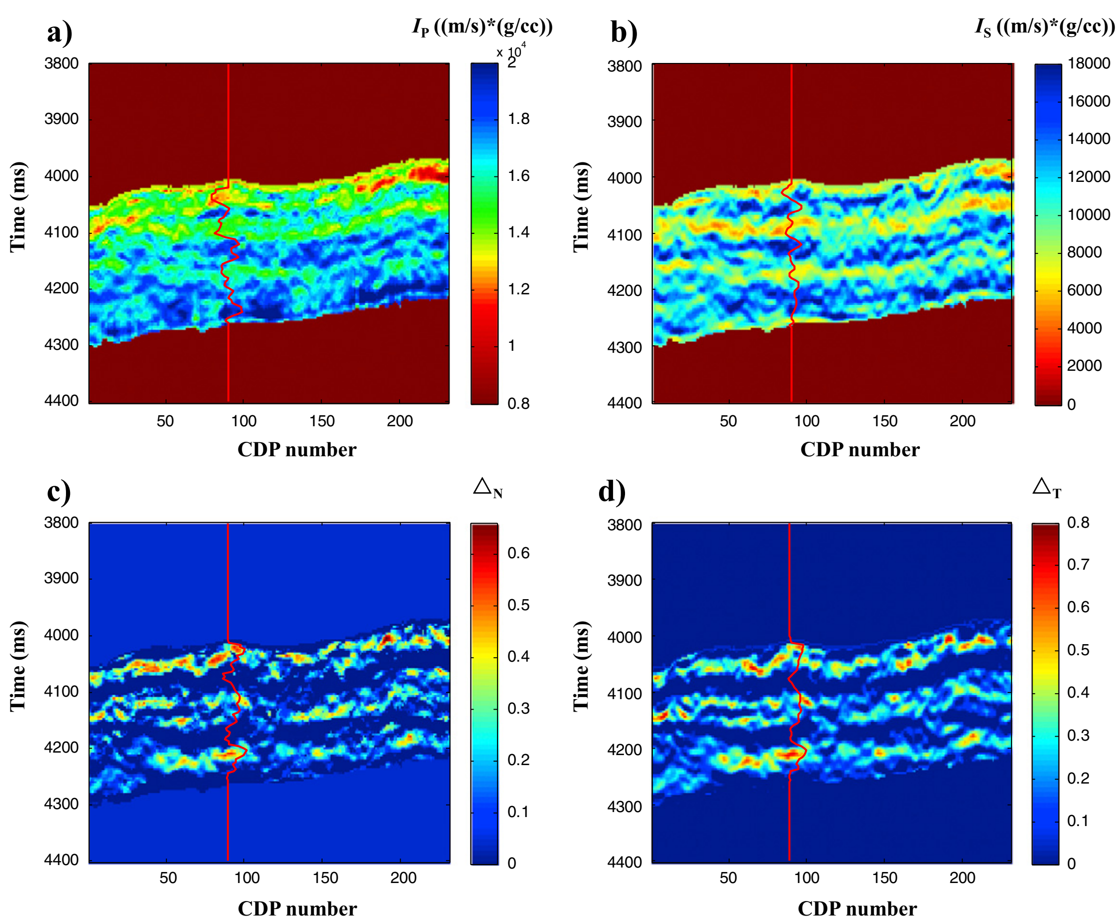

Fig. 6. Elastic and fracture weaknesses parameters estimated results. (a) P-wave impedance estimated result (b) S-wave impedance estimated result. (c) The normal weakness estimated result (d) The tangential weakness estimated result.

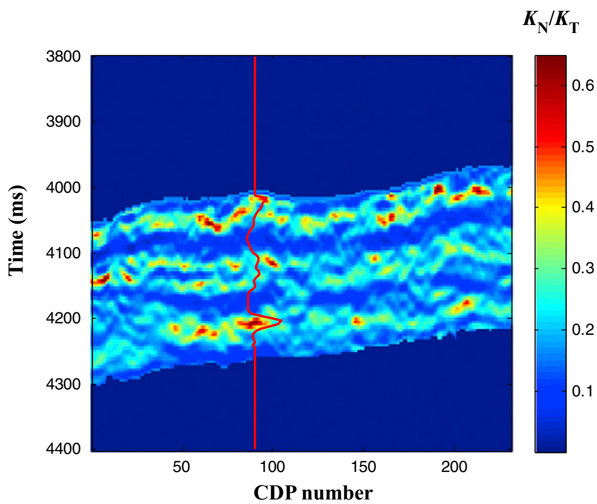

Fig. 7. Fracture fluid factor estimated value.

These results demonstrate that the method, which utilizes azimuthal EI datasets to predict the parameters of fractured reservoirs, is proved to be valid. The method is also more stable than AVAZ inversion method because it uses azimuthally incident-angle-stack gathers, which might minimize the effect of noise.

Studies show that the reflection coefficients differ obviously between different azimuths only when the incident angle is large (Gray, 2004; Hunt et al., 2010). In other words, we might obtain the unreasonable results if we purely use azimuthally P-wave seismic data. It is well known that shear-wave splitting is being in fractured rock (Li, 1998). Hence, the method which joints PP and PS wave seismic data to estimate the parameters of fractured layers may become very and very useful (Chen et al., 2014).

5. Conclusions

We utilize the relationship among Thomsen parameters, the normal and the tangential weaknesses to reformulate the HTI reflection coefficient equation in terms of P-wave impedance reflection coefficient, S-wave impedance reflection coefficient, the normal weakness and the tangential weakness. With this formulation, the normal and the tangential weaknesses can be estimated directly without using Thomsen parameters in the calculation of weaknesses parameters. Our formulation of the azimuthal EI equation can be applied to estimate P-wave impedance, S-wave impedance and weaknesses parameters. In order to enhance the stability of estimating P-wave impedance, S-wave impedance and weaknesses parameters from azimuthal EI, we present the damped least squares method. The real data is used to confirm the validity of the proposed method. The P-wave impedance, S-wave impedance and weaknesses parameters inverted results which well match the logging data show us that the azimuthal EI inversion method is more stable and reasonable. The estimated fracture fluid factor is well consistent with the drilling and rock physics analysis results, and it can help to identify the fillings in fractures.

Acknowledgements

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