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S-wave velocity self-adaptive prediction based on a variable dry rock frame equivalent model

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Abstract

Seismic velocities are important reservoir parameters in seismic exploration. The Gassmann theory has been widely used to predict velocities of fluid-saturated isotropic reservoirs at low frequency. According to Gassmann theory, dry rock frame moduli are essential input parameters for estimating reservoir velocities. A variable dry rock frame equivalent model called VDEM based on the differential effective medium (DEM) theory is constructed in this paper to obtain the dry rock frame moduli. We decouple the DEM equations by introducing variable parameters, then simplify these decoupled equations to get the equivalent dry rock frame model. The predicted dry rock frame moduli by the VDEM are in good agreement with the laboratory data. The VDEM is also utilized to predict S-wave velocity combined with Gassmann theory. A self-adaptive inversion method is applied to fit the variable parameters with the constraint of P-wave velocity from well logging data. The S-wave velocity is estimated from these inverted parameters. A comparison between the self-adaptive method and the Xu-White model on S-wave velocity estimation is made. The results corroborate that the self-adaptive method is flexible and effective for S-wave velocity prediction.

Keywords: rock physics, dry rock frame, S-wave velocity, simulated annealing algorithm

(Some figures may appear in colour only in the online journal)
of consolidation. Lee (2005) generalized the Pride model by proposing a new dry rock shear modulus. Differential effective medium (DEM) (Berryman, 1992) theory is one popular method of effective medium theory that can be used to predict the dry rock frame moduli. However, since the differential equations are coupled, it can only provide numerical solutions instead of accurate analytical expressions of the dry rock frame moduli.

In this paper, variable parameters are introduced into the dry rock frame moduli formulae to decouple the DEM equations. Thus the variable dry rock frame equivalent model called VDEM is constructed. The VDEM is firstly tested by the laboratory measured data from Han (1986). Then we combine the VDEM and Gassmann theory to build a relationship between the variable parameters and the P-wave velocity. Here we apply a simulated annealing algorithm to self-adaptively inverse the variable parameters under the constraint of the P-wave velocity from the well logging data. The S-wave velocity can be estimated from these inversion variable parameters.

### Table 1. Approximations of geometric coefficients $P$ and $Q$ for the three specific shape pores.

<table>
<thead>
<tr>
<th>Inclusion shape</th>
<th>$P$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical pores</td>
<td>$1 + \frac{3K_m - \frac{3K_m}{4\mu_m} (n-m)y}{4\mu_m}$</td>
<td>$1 + \frac{6K_m + 12\mu_m}{9K_m + 8\mu_m} + \frac{60K_m\mu_m(m-n)}{(9K_m + 8\mu_m)^2}y$</td>
</tr>
<tr>
<td>Needle-shaped pores</td>
<td>$1 + \frac{K_w}{\mu_m} + \frac{K_w}{\mu_m} (n-m)y$</td>
<td>$\left[ \frac{22}{3} + \frac{6K_w + 14\mu_m}{3K_m + \mu_m} \right] + \frac{36K_m\mu_m(m-n)}{(3K_m + \mu_m)^2}y$</td>
</tr>
<tr>
<td>Penny-shaped cracks</td>
<td>$\frac{K_m}{\alpha a \mu_m} + \frac{4\mu_m}{3K_m + \mu_m} + \frac{(3K_m + \mu_m)^3}{(n-m)y}$</td>
<td>$\left[ \frac{4(3K_m + 4\mu_m)(9K_m + 4\mu_m)}{3\alpha(3K_m + 2\mu_m)(3K_m + \mu_m)} + \frac{16(3K_m + \mu_m)^3}{\alpha(3K_m + 2\mu_m)^2(3K_m + \mu_m)^2} \right]$</td>
</tr>
</tbody>
</table>

Notes: $K_m, \mu_m$ are background bulk and shear moduli, $n, m$ indicate variable parameters, and $\alpha$ denotes the crack aspect ratio.

**Figure 1.** The changes of dry rock frame moduli with the variable parameters and porosity for three specific pore shapes. Red, blue and green lines, respectively, are spherical pores, needle-shaped pores and penny-shaped cracks. The solid line, dashed line and dotted lines denote $n-m = 0, n-m = 5$ and $n-m = 10$, respectively.
2. Methodology

2.1. Variable dry rock frame equivalent model

The differential effective medium (DEM) theory provides an insight into porous rock elastic properties, mineral composition and microstructure. Berryman (1992) gave the coupled system of ordinary differential equations for effective bulk and shear moduli

\[ (1 - y) \frac{d}{dy}[K^*(y)] = (K_2 - K^*)P(y) \]  
\[ (1 - y) \frac{d}{dy}[\mu^*(y)] = (\mu_2 - \mu^*)Q(y) \]

with the initial conditions \( K^*(0) = K_m \) and \( \mu^*(0) = \mu_m \), where, \( K_m, \mu_m \) are the bulk and shear moduli of the initial host material (phase 1) i.e. the background material, and \( K_2, \mu_2 \) are the bulk and shear moduli of the incrementally added inclusions (phase 2), while \( y \) represents the concentration of phase 2. The terms \( P \) and \( Q \) are geometric coefficients and Berryman (1995) gave coefficients \( P \) and \( Q \) for some specific shapes of inclusions. Here, the pores of rocks are regarded as the inclusions and we focus on three specific pore shapes: spherical pores, needle-shaped pores and penny-shaped cracks.

For dry rocks, the bulk and shear moduli of the inclusion materials are zeros. Then the DEM equations turn into

\[ (1 - y) \frac{d}{dy}[K^*(y)] = -K^*P(y) \]  
\[ (1 - y) \frac{d}{dy}[\mu^*(y)] = -\mu^*Q(y) \]

Generalizing the dry rock frame model of Pride (2005) and Lee (2005), we introduce the variable dry rock frame moduli \( K_d = K_m \frac{1 - y}{1 + ny} \) and \( \mu_d = \mu_m \frac{1 - y}{1 + ny} \), then put them into the geometric factors \( P \) and \( Q \) to substitute the moduli of background materials for the three specific shapes of pores, in which \( m \) and \( n \) are called variable parameters.

Take the geometric factor \( P \) of a spherical pore as an example. Berryman (1995) gave the coefficient \( P \) for a spherical pore

\[ P = \frac{K_b + 4\mu_b/3}{K_i + 4\mu_i/3} \]

where, subscripts \( b \) and \( i \) respectively refer to background and inclusion materials.

Set \( K_i = 0 \) for the dry rock frame and replace the moduli of the background materials in equation (2.5) with the variable dry rock frame moduli \( K_d = K_m \frac{1 - y}{1 + ny} \) and \( \mu_d = \mu_m \frac{1 - y}{1 + ny} \) to get

Figure 2. The application of the VDEM on laboratory data from Han et al (1986). (a): P-wave velocity versus porosity and clay content of the measured data. (b): S-wave velocity versus porosity and clay content of the measured data. (c): The comparison of dry rock frame bulk moduli between the measured data and calculated ones from the VDEM. (d): The comparison of dry rock frame shear moduli between the measured data and calculated ones from the VDEM.
and \( K \) is the bulk modulus. To obtain the equivalent model, we can set

\[
P = 1 + \frac{3K_m}{4\mu_m} y + \frac{6K_m}{4\mu_m} (n - m) y
\]

(2.6)

Applying a first-order Taylor series around \( y = 0 \), we can obtain the approximate coefficients

\[
P \approx 1 + \frac{3K_m}{4\mu_m} [1 + (n - m)y]
\]

(2.7)

Similarly, the approximations of the three specific geometric coefficients \( P \) and \( Q \) are shown in Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Quartz</th>
<th>Clay</th>
<th>Calcite</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K ) (GPa)</td>
<td>37</td>
<td>21</td>
<td>76.8</td>
<td>2.2</td>
</tr>
<tr>
<td>( \mu ) (GPa)</td>
<td>44</td>
<td>7</td>
<td>32</td>
<td>0</td>
</tr>
</tbody>
</table>

Next, rewrite these approximated coefficients into the unified form \( P = P_1 + P_2 y, Q = Q_1 + Q_2 y \), where \( P_1, Q_1 \) and \( P_2, Q_2 \) are respectively constant term coefficients and first-order term coefficients.

Then, replace \( P^{\phi + y}, Q^{\phi + y} \) in equation (2.3) and equation (2.4) with the approximated coefficients \( P, Q \) to obtain

\[
\frac{d[\ln K^{\phi}]}{K^\phi} = - \frac{P_1 + P_2 y}{1 - y} dy
\]

(2.8)

\[
\frac{d[\ln \mu_\phi]}{\mu_\phi} = - \frac{Q_1 + Q_2 y}{1 - y} dy
\]

(2.9)

Integrating \( y \) from 0 to \( \phi \), we can get

\[
\ln K_m [K^{\phi}] = [\ln (1 - y)^{P_1 + P_2 y^2}]
\]

(2.10)

The variable dry rock frame equivalent moduli are obtained

\[
K_\phi(y) = K_m (1 - \phi)^{P_1 + P_2 y^2 Q_1 + Q_2 y^2}
\]

(2.11)

\[
\mu_\phi(y) = \mu_m (1 - \phi)^{Q_1 + Q_2 y^2}
\]

(2.12)

where, \( K_m, \mu_m \) are the bulk and shear moduli of the initial host material, \( \phi \) is the porosity, \( P_1, Q_1 \) and \( P_2, Q_2 \) are respectively the constant term coefficients and first-order term coefficients of the geometric coefficients \( P, Q \).

We can see from Table 1 that the differences between variable parameters \( m \) and \( nd \) dominate the value of the geometric coefficients, thus they control the results of dry rock frame equivalent moduli. The variable parameter \( m \) of the dry-frame bulk moduli is similar to the consolidation parameter of the Pride (2005) and Lee (2005) dry rock model. Then the range \( 0 \leq n - m < 20 \) of the variable parameters difference is recommended in practical applications. Especially, when \( n = m \), the dry rock frame moduli are identical to those of Keys and Xu (2002) i.e. Keys and Xu’s dry rock frame approximations are special cases of the variable dry rock frame equivalent model.

The differences of the variable parameters can be treated as free factors to fit for estimating the dry rock frame moduli or matching the observed velocities. The changes of the dry rock frame moduli with porosity for the three specific shape pores are given in Figure 1 (background moduli \( K_m = 37 GPa, \mu_m = 44 GPa \)). The figure shows that both the dry rock frame bulk and shear moduli decrease with the increase of porosity. The dry rock frame with spherical pores has larger bulk and shear moduli than rocks containing...
needle-shaped pores and penny-shaped cracks of the same porosity and difference of variable parameters. Besides, the dry rock frame bulk moduli decrease with the increase of the \( n - m \) value, which are contrary to the change of the shear moduli.

2.2. Gassmann theory

Gassmann (1951) equations can be used to calculate the fluid-saturated rock moduli from the dry rock frame moduli, solid matrix moduli, pore fluid bulk modulus and porosity for an isotropic reservoir at low frequency:

\[
K_s = K_d + \left( 1 - \frac{K_d}{K_m} \right) ^ 2 \left( \frac{\phi}{K_f} + \frac{1 - \phi}{K_m} - \frac{K_d}{K_m} \right) \tag{2.14}
\]

\[
\mu_s = \mu_f \tag{2.15}
\]

where, \( K_s, K_d, K_m \) and \( K_f \) are the bulk moduli of fluid-saturated rock, the dry rock frame, the matrix and pore fluid, respectively. \( \mu_s, \mu_f \) are the shear moduli of fluid-saturated rock and the dry rock frame, \( \phi \) denotes the total porosity.

The rock velocity can be computed after the fluid-saturated rock moduli are obtained

\[
V_p = \sqrt{\frac{K_s + 4\mu_s}{3}} \rho \tag{2.16}
\]

\[
V_s = \sqrt{\frac{\mu_s}{\rho}} \tag{2.17}
\]

where, \( V_p, V_s \) are respectively P and S-wave velocities and \( \rho \) is the bulk density of fluid-saturated rock.

3. Applications

3.1. Dry rock frame moduli estimation

We use 75 fluid-saturated sandstone samples measured by Han et al. (1986) at 40 MPa to test the VDEM. The porosity of these samples ranges from 2% to 30% and the volume of the clay content is between 0 and 50%. Figures 2 (a)–(b) show the relations between the rock velocities and the porosity of these samples. We get the rocks matrix moduli by the method proposed by Han et al. (1986). The fluid-saturated rock moduli are obtained from the measured velocity and bulk density data. We put these values into Gassmann equations to calculate the dry rock frame moduli, which are treated as measured data to compare with the VDEM results. We assume the pores of the rocks are penny-shaped cracks with the aspect ratio of 0.03 and set \( n = m = 2 \). The comparison results are shown in figures 2 (c)–(d). We can see that most of the VDEM calculation results are in good agreement with the measured ones. Some deviations might be caused by the assumption of the single pore type for all these samples.

3.2. S-wave velocity self-adaptive prediction

We combine the VDEM with Gassmann equations to build the relationship between the P-wave velocity and the
variable parameters. The S-wave velocity can be predicted by self-adaptively inverting these variable parameters under the constraint of the P-wave velocity from the well logging data. Figure 3 gives some of the well logging data of a sandstone reservoir. The figure shows that the depth ranges from 3500 m to 3600 m and the reservoir is mainly filled with water.

Firstly, the Voigt-Reuss-Hill average (Hill, 1952) is applied to estimate the matrix bulk and shear moduli. The average is expressed as

\[ M_{VRH} = \frac{M_V + M_R}{2} \]  

where

\[ M_V = \sum_{i=1}^{N} f_i M_i \]  

\[ \frac{1}{M_R} = \sum_{i=1}^{N} \frac{f_i}{M_i} \]  

The terms \( f_i \) and \( M_i \) are the volume fraction and modulus of the \( i \)th component respectively. \( M_V \) and \( M_R \), respectively, denote the Voigt upper bound and the Reuss lower bound. The moduli of the minerals and fluid used in this well are given in table 2.

Secondly, we imitate the Xu-White model (1995) to divide the total porosity into compliant shale pores and stiff sandstone pores, represented by penny-shaped cracks and needle-shaped pores. Assume that \( c \) and \( s \) are respectively proportional to clay fractional volume \( c \) and sand fractional volume \( s \), where \( c + s = 1 \). Combine Kuster–Toksöz (K–T) equations (1974) and DEM theory by introducing variable parameters to estimate the dry rock frame moduli. The K–T equations for the dry rock frame moduli are

\[ K_d - K_m = (K_i - K_m)\frac{3K_d + 4K_m}{3K_m + 4K_m} \sum_{i=1}^{N} \frac{f_i P^{mi}}{M_i} \]  

\[ \mu_d - \mu_m = (\mu_i - \mu_m)\frac{3\mu_d + 4\mu_m}{5\mu_m + 4\mu_m} \sum_{i=1}^{N} \frac{f_i Q^{mi}}{M_i} \]  

where, the subscripts \( c, s \) respectively refer to compliant shale pores and stiff sandstone pores.

However, these equations require \( \phi/\alpha \ll 1 \), which is usually difficult to satisfy. According to Berryman (1992), the
DEM equations converged to ordinary differential equations when the incremental porosity is small enough. Thus the K–T equations become

\[
(1 - \phi) \frac{d\bar{K}}{d\phi} = (K_i - K_m) \sum_{j=x,c} \nu_j \bar{P}_j^{mi} \tag{3.6}
\]

\[
(1 - \phi) \frac{d\bar{\mu}}{d\phi} = (\mu_i - \mu_m) \sum_{j=x,c} \nu_j \bar{Q}_j^{mi} \tag{3.7}
\]

Respectively, substitute \( \bar{P}_j^{mi} \) and \( \bar{Q}_j^{mi} \) in equation (3.6) and equation (3.7) with the approximations of coefficients \( P \) and \( Q \) for needle-shaped pores and penny-shaped cracks. After that, integrate these equations to obtain the moduli of the dry rock frame

\[
K_d(\phi) = K_m(1 - \phi)^{P_i + P_1 e^{\phi_p}} \tag{3.8}
\]

\[
\mu_d(\phi) = \mu_m(1 - \phi)^{\mu_1 + \mu_2 e^{\phi_2}} \tag{3.9}
\]

where, \( P_1 = \nu_1 P_1 + \nu_2 P_2 + \nu_3 P_3, P_2 = \nu_1 P_2 + \nu_2 P_2 + \nu_3 P_5, Q_1 = \nu_1 Q_1 + \nu_2 Q_2 + \nu_3 Q_3 \), and \( Q_2 = \nu_1 Q_2 + \nu_2 Q_2 + \nu_3 Q_3 \), subscripts \( c \), \( s \) respectively denote penny-shaped cracks and needle-shaped pores. These equations imply that the order in which compliant and stiff pores are added to rock has no influence on the dry rock frame moduli.

Finally, we utilize Gassmann equation (2.14) and equation (2.15) to calculate the fluid-saturated rock moduli with the variable parameters. After that, we predict the S-wave velocities from the well logging data.

To evaluate the S-wave prediction results, we compare our self-adaptive method with the Xu-White model (1996) with the same input data. The comparison results are shown in figure 5. We can see from the figure that the self-adaptively predicted results match well with the measured data and the relative errors of P-wave and S-wave velocities have an approximately normal distribution, shown in the histogram. The P-wave and S-wave velocities estimated by the Xu-White model generally agree with the well log data, but fail at the intervals where the reservoir parameters change dramatically (shown by the black arrows). The test results elaborate that the self-adaptive method is better at S-wave velocity prediction than the Xu-White model of this reservoir. The reason might be that the self-adaptive method introduces the variable parameters, which are changeable with depth and adaptive for the complex reservoir conditions.

4 Conclusions

In this paper, we constructed a variable dry rock frame equivalent model (VDEM) for three specific shapes of inclusions based on differential effective medium theory. The variable parameters introduced by the dry rock frame moduli can be treated as factors that represent the pressure, the degree of consolidation and other in situ geological conditions that might affect the dry rock frame moduli. Additionally, the VDEM has the advantage of considering the effect of the inclusion geometry shape on the dry rock frame moduli. These features broaden the applicability of the VDEM.

The applications of the VDEM to estimate the dry rock frame moduli are in good agreement with laboratory data, which confirm the VDEM is effective at estimating the dry rock frame moduli. Then the VDEM is combined with the Gassmann theory to predict the S-wave velocity of isotropic rocks. The simulated annealing algorithm is used to self-adaptively inverse the variable parameters by minimizing the error between the calculated P-wave velocities and the measured ones from the well logging data. The S-wave velocity is predicted from those inversion parameters. The application of the self-adaptive method to a sandstone well’s logging data shows that the predicted S-wave velocities match well with the measured ones, which proves our self-adaptive S-wave velocity predicting method is practical and feasible. Additionally, the comparisons between the self-adaptive method and the Xu-White model on S-wave velocity prediction demonstrate that the self-adaptive method is more flexible and is better suited for a complex reservoir.

Acknowledgments

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